Oxford Cambridge and RSA

Tuesday 4 June 2024 – Afternoon

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq \left(k + \tfrac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r\, p^r q^{n-r}$ where $q = 1 - p$

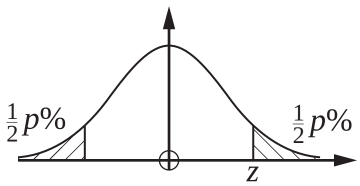
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A (25 marks)

1 A student states that $1+x^2 < (1+x)^2$ for all values of x.

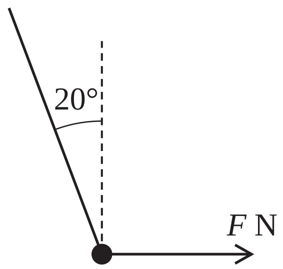
Using a counter example, show that the student is wrong. **[2]**

2 A car of mass 1400 kg pulls a trailer of mass 400 kg along a straight horizontal road. The engine of the car produces a driving force of 6000 N. A resistance of 800 N acts on the car. A resistance of 300 N acts on the trailer. The tow-bar between the car and the trailer is light and horizontal.

(a) Draw a force diagram showing all the horizontal forces on the car and the trailer. **[2]**

(b) Calculate the acceleration of the car and trailer. **[3]**

3 A particle hangs at the end of a string. A horizontal force of magnitude F N acting on the particle holds it in equilibrium so that the string makes an angle of 20° with the vertical, as shown in the diagram. The tension in the string is 12 N.



(a) Find the value of F. **[2]**

(b) Find the mass of the particle. **[3]**

4 The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are defined by $\mathbf{v}_1 = 2a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v}_2 = b\mathbf{i} - 3\mathbf{j}$ where a and b are constants.

Given that $3\mathbf{v}_1 + \mathbf{v}_2 = 22\mathbf{i} - 9\mathbf{j}$, find the values of a and b. **[4]**

5 **(a)** Make y the subject of the formula $\log_{10}(y-k) = x\log_{10}2$, where k is a positive constant. **[2]**

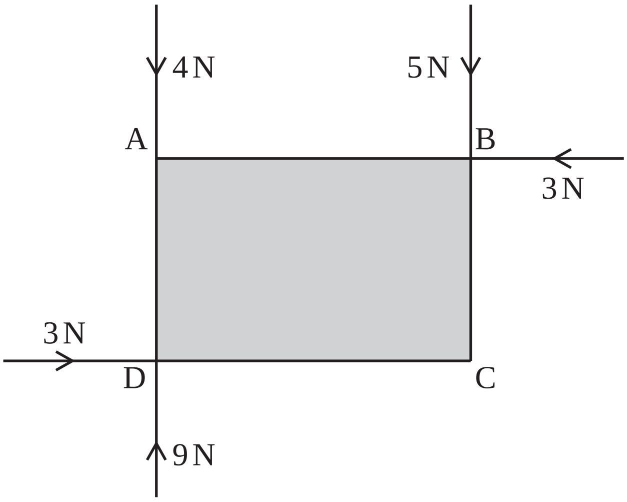
(b) Sketch the graph of y against x. **[3]**

6 Given that $f(x) = 2x^2 + 3$, show from first principles that $f'(x) = 4x$. **[4]**

Section B (75 marks)

7 A rectangular book ABCD rests on a smooth horizontal table. The length of AB is 28 cm and the length of AD is 18 cm. The following five forces act on the book, as shown in the diagram.

- 4 N at A in the direction AD
- 5 N at B in the direction BC
- 3 N at B in the direction BA
- 9 N at D in the direction DA
- 3 N at D in the direction DC



(a) Show that the resultant of the forces acting on the book has zero magnitude. **[2]**

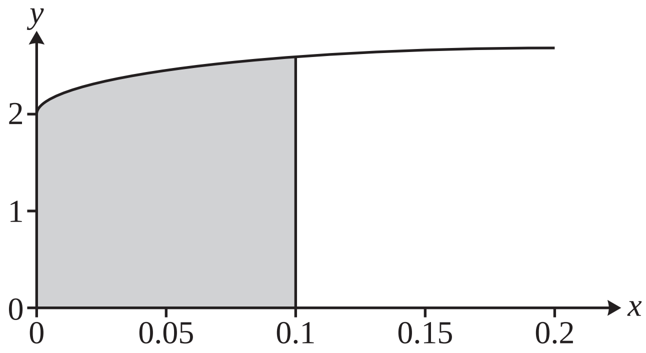
(b) Find the total moment of the forces about the centre of the book. Give your answer in N m. **[3]**

(c) Describe how the book will move under the action of these forces. **[2]**

8 The equation of a curve is $y = \sqrt{\sin 4x} + 2\cos 2x$, where x is in radians.

(a) Show that, for small values of x, $y \approx 2\sqrt{x} + 2 - 4x^2$. **[2]**

The diagram shows the region bounded by the curve $y = \sqrt{\sin 4x} + 2\cos 2x$, the axes and the line $x = 0.1$.



(b) **In this question you must show detailed reasoning.**

Use the approximation in part **(a)** to estimate the area of this region. **[4]**

9 A child throws a pebble of mass 40 g vertically downwards with a speed of $6\,\mathrm{m\,s^{-1}}$ from a point 0.8 m above a sandy beach.

(a) Calculate the speed at which the pebble hits the beach. **[2]**

The pebble travels 3 cm through the sand before coming to rest.

(b) Find the magnitude of the resistance force of the sand on the pebble, assuming it is constant. Give your answer correct to **3** significant figures. **[5]**

10 Zac is measuring the growth of a culture of bacteria in a laboratory. The initial area of the culture is $8\,\text{cm}^2$. The area one day later is $8.8\,\text{cm}^2$.

At first, Zac uses a model of the form $A = a + bt$, where $A\,\text{cm}^2$ is the area t days after he begins measuring and a and b are constants.

(a) Find the values of a and b that best model the initial area and the area one day later. **[2]**

(b) Calculate the value of t for which the model predicts an area of $15\,\text{cm}^2$. **[1]**

(c) Zac notices the area covered by the culture increases by 10% each day.

Explain why this model may not be suitable after the first day. **[1]**

Zac decides to use a different model for A. His new model is $A = Pe^{kt}$, where P and k are constants.

(d) Find the values of P and k that best model the initial area and the area one day later. **[3]**

(e) Calculate the value of t for which the area reaches $15\,\text{cm}^2$ according to this model. **[2]**

(f) Explain why this model may not be suitable for large values of t. **[1]**

11 The first three terms of a geometric sequence are $5k-2,\ 3k-6,\ k+2$, where k is a constant.

(a) Show that k satisfies the equation $k^2 - 11k + 10 = 0$. **[3]**

(b) When k takes the smaller of the two possible values, find the sum of the first 20 terms of the sequence. **[3]**

(c) When k takes the larger of the two possible values, find the sum to infinity of the sequence. **[2]**

12 In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the x- and y-directions respectively.

The velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ of a particle is given by $\mathbf{v} = 3\mathbf{i} + (6t^2 - 5)\mathbf{j}$. The initial position of the particle is $7\mathbf{j}\,\text{m}$.

(a) Find an expression for the position vector of the particle at time t s. **[4]**

(b) Find the Cartesian equation of the path of the particle. **[2]**

13 The curve with equation $y = px + \frac{8}{x^2} + q$, where p and q are constants, has a stationary point at (2, 7).

(a) Determine the values of p and q. **[5]**

(b) Find $\frac{d^2y}{dx^2}$. **[1]**

(c) Hence determine the nature of the stationary point at (2, 7). **[2]**

14 A man runs at a constant speed of $4\,\mathrm{m\,s^{-1}}$ along a straight horizontal road. A woman is standing on a bridge that spans the road. At the instant that the man passes directly below the woman she throws a ball with initial speed $u\,\mathrm{m\,s^{-1}}$ at $\alpha°$ above the horizontal. The path of the ball is directly above the road. The man catches the ball 2.4 s after it is thrown. At the instant the man catches it, the ball is 3.6 m below the level of the point of projection.

(a) Explain what it means that the ball is modelled as a particle. **[1]**

(b) Find the vertical component of the ball's initial velocity. **[2]**

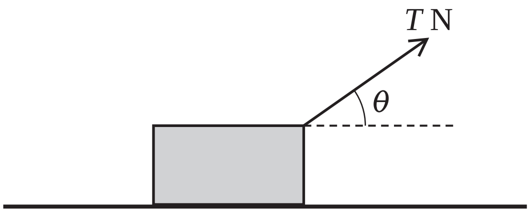
(c) Find each of the following.
- The value of u
- The value of α **[4]**

15 The circle $x^2 + y^2 + 2x - 14y + 25 = 0$ has its centre at the point C. The line $7y = x + 25$ intersects the circle at points A and B.

Prove that triangle ABC is a right-angled triangle. **[9]**

Turn over for question 16

16 A block of mass m kg rests on rough horizontal ground. The coefficient of friction between the block and the ground is μ. A force of magnitude T N is applied at an angle θ radians above the horizontal as shown in the diagram and the block slides without tilting or lifting.



(a) Show that the acceleration of the block is given by $\frac{T}{m}\cos\theta - \mu g + \frac{T}{m}\mu\sin\theta$. **[4]**

For a fixed value of T, the acceleration of the block depends on the value of θ. The acceleration has its greatest value when $\theta = \alpha$.

(b) Find an expression for α in terms of μ. **[3]**

END OF QUESTION PAPER

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